# Leonhard Euler

His Life and His Faith

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# Leonhard Euler (1707–1783)

## 1 Introduction

When I was a mathematics student in college there were many times when I encountered the name Euler — Euler equations, Euler functions, Euler theorems, etc. Unfortunately, I didn’t take the time to investigate the man behind the name. It turns out that Leonhard Euler was one of the greatest mathematicians and scientists of all time. Mathematics historian Morris Kline ranks him at the highest level with Archimedes, Newton, and Gauss [5]. Euler made significant contributions to nearly every branch of mathematics — analysis (calculus, differential equations, calculus of variations), algebra, number theory, complex variables, Euclidean and differential geometry, topology, graph theory, and combinatorics. He also made considerable advances and discoveries in many branches of physics — mechanics, astronomy, electricity and magnetism, light and color, hydraulics, optics, acoustics, and elasticity. Science historian Clifford Truesdell regarded him as the dominating theoretical physicist of the eighteenth century [7]. If that wasn’t enough, he also wrote landmark papers concerning the building and navigation of ships, artillery theory, and the foundations of actuarial science. The famous mathematician Pierre Simon Laplace made the statement

> *Lisez Euler, lisez Euler, c’est notre matre  tous.*
> (Read Euler, read Euler, he is our master in everything.)

In addition to his many discoveries, he also clarified many existing areas of mathematical science and established much of the notation we use today. For example, he established the use of

- $e$ for the base of the natural logarithm
- $\pi$ for the ratio of the circumference to the diameter of a circle
- $f(x)$ for functional value
- $\sin x$ and $\cos x$ for values of the sine and cosine functions
- $i$ for the imaginary unit
- $\Sigma$ for summation
- $\Delta$ for finite difference

The following terminology in mathematics and physics is associated with Euler: Euler angles, Euler circuits, the Euler $\phi$-function, the Euler-Lagrange equation, Euler's identity, Eulerian mechanics, the Euler-Maclaurin summation formula, Euler's addition theorem for elliptic integrals, the Euler-Descartes formula, and many more.

Euler was certainly one of the most prolific scientific authors of all time. During his lifetime he had more than 500 books and articles published. An additional 400 of his manuscripts were published after his death. Historian Clifford Truesdell estimated that approximately one third of all publications in the fields of mathematics, theoretical physics, and engineering mechanics between the years 1725 and 1800 were authored by Euler [7]. Today mathematicians such as Euler are not well known by the public. However, in Euler's day, mathematics was considered the highest form of knowledge and he was better known by the general public then such literary and music greats as Swift and Bach. His collected works, entitled *Euleri Opera Omnia*, is contained in 80 volumes, many of which exceed 500 pages. A picture of this collection is shown in Figure 1.


Figure 1: Opera Omnia at Euler-Archiv in Basel

The works in this collection are distributed as follows:

| | |
|---|---|
| 40% | Advanced Algebra, Number theory, and Mathematical Analysis |
| 28% | Mechanics and Physics |
| 18% | Geometry |
| 11% | Astronomy |
| 2% | Artillery, Architecture, and Naval science |
| 1% | Various other subjects |

Euler was well known for the clarity of his exposition. Clifford Truesdell [7] says of Euler's writings

> *It was Euler who first in the western world wrote mathematics openly, so as to make it easy to read. He taught his era that the infinitesimal calculus was something any intelligent person could learn, with application, and use. He was justly famous for his clear style and for his honesty to the reader about such difficulties as there were.*

In many cases Euler arrived at his results through an inductive process involving the consideration of many special cases. Unlike most authors Euler did not hide this path of discovery from his readers. He described in detail the examples and reasoning that led him to the result. Thus, the reader is exposed to the thought process of a master.

Euler was gifted with an extraordinary memory and an ability to perform complicated calculations in his head. He could recite from memory the epic poem Aeneid in Latin (approximately 400 pages) and could tell you the first and last line on each page of the original text he used. Once when he had insomnia he calculated in his head the first six powers of all the numbers from 1 to 100 and committed this table to memory. He frequently amused his friends by recalling some of these results. At another time two of his students performed a difficult calculation involving the sum of a power series up to the seventeenth term and their answers differed in the fifth digit. They went to Euler to settle their dispute. Euler performed the calculation in his head and not only got the correct answer, but was able to point out where each of the students had erred. This ability served him well during the last 17 years of his life when he was almost totally blind. Nearly one half of his works were composed during this final period of his life. The French mathematician and physicist Franois Arago made this statement

> *Euler calculated without apparent effort, as men breathe, or as eagles sustain themselves in the wind.*

Euler was universally admired, not only for his genius, but also for his character. He seemed to have had no desire to advance his own career at the expense of others. Clifford Truesdell wrote [7]

> *He was exceptionally generous, never once making a claim of priority and in some cases actually giving away discoveries that were his own. He was the first to cite the work of others in what is now regarded as the just way, that is, so as to acknowledge their worth.*

Euler was one of the principal developers of what is now called the Calculus of Variations. After spending much time and effort in developing this area he received a letter from a young French mathematician Joseph-Louis Lagrange proposing a new analytic approach for solving variational problems. Euler immediately recognized the superiority of Lagrange's approach and set about publicizing Lagrange's discovery, giving him full credit. He even refused to publish his own work on the subject until after Lagrange's work was published. They became lifelong friends. Euler also delayed publishing his treatise on hydrodynamics until his friend Daniel Bernoulli had published his.

Euler fathered 13 children, only five of which reached adolescence. His children provided him with 38 grandchildren. He was devoted to his family and said that much of his best work was accomplished with children playing at his feet and crawling on his lap. Euler was a committed Christian and frequently expressed awe at the works of the Creator. Euler was particularly impressed by the design of the eye. Here is one statement that he made concerning the eye

> *though we are very far short of a perfect knowledge of the subject, the little we do know of it is more than sufficient to convince us of the power and wisdom of the Creator. …We discover in the structure of the eye perfections which the most exalted genius could never have imagined*

Concerning the calculus of variations he wrote

> *For since the fabric of the universe is most perfect and the work of a most wise creator, nothing at all takes place in the universe in which some rule of the maximum or minimum does not appear.* — Leonhard Euler, Methodus Inveniendi Uneas Curvas, 1st addition, art. 1 (1744).

Each night, until he lost his sight, he read to his family from the Bible and discussed with them the meaning of what was read. His faith was often ridiculed by Enlightenment philosophers such as Voltaire. In defense of his faith he wrote the document *Defense of the Divine Revelation against the Objections of the Freethinkers* [Leonhardi Euleri Opera Omnia, Ser. 3, Vol. 12].

## 2 Biographical Sketch

Leonhard Euler was Born in Basel on April 15, 1707. When he was one the family moved to Riehen, a town near Basel, where his father became a pastor. His father taught him elementary mathematics as a child. He entered the University of Basel at age 14. Euler's childhood home is shown in Figure 2a and the University of Basel is the building in the foreground of Figure 2b. His fathers goal was for him to study theology and become a pastor. While at the university he met the famous mathematician Johann Bernoulli. In Eulers own words

*. . . I soon found an opportunity to be introduced to a famous professor Johann Bernoulli. . . . True, he was very busy and so refused flatly to give me private lessons; but he gave me much more valuable advice to start reading more difficult mathematical books on my own and to study them as diligently as I could; if I came across some obstacle or difficulty, I was given permission to visit him freely every Sunday afternoon and he kindly explained to me everything I could not understand . . .*

Figure 2a: Euler's childhood home in Riehen next to father's church

Figure 2b: The University of Basel

In 1723 (at age 16) Euler completed his Master's degree in philosophy. In his thesis he compared and contrasted the philosophical ideas of Descartes and Newton. He began the study of theology in the fall of 1723. However, he was still strongly attracted to mathematics. Johann Bernoulli finally convinced his father to allow him to change to mathematics. In spite of this change of vocation, his Christian faith remained strong throughout his life. He graduated in 1726. While in school he became close friends with the Bernoulli brothers Johann II, Daniel, and Nicolaus (sons of Johann). He was especially close to Daniel. In 1727 he won second place in the contest for the Grand Prize of the Paris Academy with an article on the best placement of masts on a ship. This was amazing since he, at this time, had never seen a ship. He would later win the Grand Prize twelve times.

The Bernoulli brothers, Daniel and Nicolaus, went to Russia and joined the Saint Petersburg Academy of Sciences in 1725, two years after it had been founded by Catherine I the wife of Peter the Great. Nicolaus Bernoulli could not handle the harsh Russian climate, and died prematurely in 1726. Euler was invited to Saint Petersburg in 1727. He was originally recruited for a position in the physiology division, but through the requests of Daniel Bernoulli and Jakob Hermann, Euler was appointed to the mathematical-physical division. The Saint Petersburg Academy of Sciences is shown in Figure 3.

Figure 3: The Saint Petersburg Academy of Sciences

The Saint Petersburg Academy of Sciences was established in order to bring Russia up to the same level in Science as other countries in Europe. Many talented scientists from other countries were imported to staff the Academy. They were given great freedom to pursue their research. Contrary to most of the foreign members of the Academy Euler quickly learned to read, write, and speak the Russian language. In 1732 the youngest Bernoulli brother, Johann II, came to Saint Petersburg and a year later he and Daniel decided to return to Switzerland. Daniel was offered a professorship in both Anatomy and Botany at the University of Basel. Their departure saddened Euler as they were his closest friends. Euler took over Daniel's chair in mathematics at the Saint Petersburg Academy. In December of 1733 Euler married a native Swiss of the same age, Catherine Gsell, the daughter of a painter from the St Petersburg Gymnasium. Euler and his family remained in Russia until 1741. While in Russia Euler solved the 91-year-old Basel problem in infinite series, namely, finding an exact sum of the infinite series $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$. In the process of solving this problem he developed what is now called the Euler-Maclaurin summation formula. This summation formula often improved the convergence rate of convergent infinite series and sometimes converted divergent series to convergent ones. He also made valuable contributions to number theory, the theory of partitions, mechanics, acoustics, and naval science. Toward the end of his stay in Russia, Euler lost the sight in one eye, possibly as the result of a fever.

Political unrest in Russia led to a tense working environment at the St. Petersburg Academy, prompting Euler to accept an invitation from Frederick the Great of Prussia to join his Royal Academy at Berlin in 1741. A drawing of the Academy is shown in Figure 4. Frederick was enamored with everything French. In particular, he was drawn to the famous French philosopher Voltaire who made frequent visits to Frederick's court. Both Frederick and Voltaire ridiculed Euler's Christian faith. However, Euler was too famous as a mathematician for Frederick to replace him. Frederick asked Euler to tutor one of his nieces in the physical sciences and Euler agreed. This resulted in a series of over 200 letters that have been collected under the title *Letters of Euler to a German Princess on Different Subjects in Natural Philosophy*. These letters explained in laymen's language the basic concepts of physics as well as Euler's views on philosophy and theology. These

Figure 4: Drawing of the Berlin Academy of Sciences

letters were later published by the Saint Petersburg Academy in two large illustrated volumes. This work was tremendously popular with the general public and went through many editions. It was translated into French, English, German, Swedish, Italian, Danish, and Spanish. In the 25 years that Euler spent in Berlin he made important discoveries in the Calculus of Variations, established Eulers Identity for Complex Numbers, produced two treatises in analysis (*Introduction to the Analysis of the Infinite* in 1748 and *Foundations of Differential Calculus, with Applications to Finite Analysis and Series* in 1755), and explored key concepts in algebra such as solutions of polynomial equations and the Fundamental Theorem of Algebra.

In 1766 he accepted the invitation of Catherine the Great to return to the Saint Petersburg Academy. Shortly after returning, he lost sight in his remaining good eye following cataract surgery. He was virtually blind for the last seventeen years of his life. After losing the ability to see he commented

> *Now I will have less distraction.*

In spite of his handicap, Euler's productivity never declined. He used his exceptional memory and the ability to perform calculations in his head to compose close to 400 additional manuscripts. He either wrote his results in large letters on a slate or dictated them to his sons or a secretary. In 1771 a fire destroyed Euler's home and he barely escaped with his life. Most of his manuscripts were rescued. In 1773 he lost his wife of 40 years. He married his wife's half-sister three years later. His two adult daughters died just prior to his own death. He died of a stroke on September 18, 1783 while engaged in his work. A picture of his tomb is shown in Figure 5. At his death the St. Petersburg Academy Journal had such a massive backlog of his work to publish that it took them another 48 years to complete.

During his second tour in St. Petersburg he produced influential works in integral calculus, algebra, dioptrics, navigation, and the theory of lunar motion.

Figure 5: Euler's tomb in Saint Petersburg.

## 3 Euler's Religious Faith

I think it is clear that Euler's religious faith played a big part in both his professional life and in his family life. We can see this by the way he interacted with others and by the way he dealt with adversity. Our knowledge of his religious beliefs comes from two of his publications: *Letters of Euler to a German Princess on Different Subjects in Natural Philosophy* and *Defense of the Divine Revelation against the Objections of the Freethinkers*. We will refer to these as [Letters] and [Defense].

Concerning the scriptures, he wrote

> *The Holy Scripture not only provides those who are seriously concerned with the improvement of their hearts with the most powerful means to that end, but that it also leads them in time to a greater knowledge of God*. [Defense, XXIII]

Concerning God's sovereignty and man's free will, he wrote

> *We must acknowledge the government and providence of God, who having from all eternity (foreseen) all the counsels, the projects, and the voluntary actions of men, arranged the corporeal world in such a manner, that it brings about, at all times, circumstances which cause these enterprises to fail, or to succeed, according as His infinite wisdom judges to be most fit. God thus remains absolute sovereign of all events, notwithstanding the liberty of men, all whose actions, though free, are, from the beginning, part of the plan which God intended to execute, when He created this universe*. [Letters, p. 383]

Concerning divine providence he wrote

> *To [Scripture] belongs in particular the doctrine of both general as well as particular divine providence, through which we can recognize that we can never get into any circumstance where God did not expressly place us according to his infinite wisdom and goodness, and can come to the firm assurance that not a single hair may fall from our head without the will of our heavenly Father. Now if only we were to ponder this doctrine with the appropriate attention and apply it to ourselves, we would submit to the will of God under all circumstances without difficulty and even with pleasure, and in this way attain true happiness.* [Defense, XXVIII]

Concerning Christ, he wrote

> *It is therefore a settled truth that Christ is risen from the dead: since this is such a marvel, which could only be performed by God alone, it makes it impossible to cast any doubt on the divine sending of Christ into this world. Consequently, the doctrine of Christ and of his apostles is divine and since it is directed toward our true happiness, we can therefore believe with the strongest confidence all the promises which have been made in the gospel regarding this life as well as the one to come, and view the Christian religion as a divine work aiming at our spirituality. But it is not necessary to elaborate further on all this, since each one who is convinced only once of the resurrection of Christ cannot doubt any further the divinity of Holy Scripture.* [Defense, XXXVI]

For further information on Euler's theology read *The God Fearing Life of Leonhard Euler* by Dale L. McIntyre [6].

## 4 A Brief Overview of Some of Euler's Scientific Work

In this section we will look at some of the areas in which Leonhard Euler made significant contributions. However, Euler's interests were so broad and the number of his publications so large that it is impossible to cover them all. Hopefully, what I have presented will give you some idea of the importance of his scientific work.

### 4.1 Dioptrics

Different colors, having different wavelengths, are refracted differently by a lens. The image produced by a spherical lens has fringes of rainbow like color. Euler investigated the cause of this distortion. In Memoires of the Berlin Academy in 1747, 1752, and 1753 he showed how to eliminate chromatic distortion of images by lenses. He showed that it was necessary to use multiple lenses with different refractive properties. The motivation for his work came from looking at the human eye that doesn't have this problem. Newton had tried to do this using two lenses separated by a water layer, but did not succeed. Newton claimed that it was not possible to correct for chromatic distortion in lenses and he began looking at reflective type telescopes using mirrors. Euler's theory predicted that correction should be possible using lenses of the type Newton used. The Englishman Dolland performed experiments to test Euler's theory. He actually believed going in to the experiment that Newton was correct, but his results confirmed Euler's theory. In 1757 Dolland was able to construct a lens system made of two types of glass that was free of chromatic aberration. Such a lens is called an achromatic lens. This is a good example of where a theoretical discovery has led to a practical application.

Euler's three large volumes entitled *Dioptrica* (1761–1771) provide a complete theory of the way light waves act in lenses. The first volume presents the general theory while the second and third volumes examine manufacturing processes for producing eye glasses, telescopes and microscopes.

## 4.2 Naval Science

Euler was very interested in the construction of ships and in navigation. His major work on this topic was *Scientia Navalis* published in 1749. It consisted of two large volumes. Some of the topics covered were

- the equilibrium of ships;
- the stability of equilibrium;
- the oscillations of ships;
- inclination under the influence of arbitrary forces;
- the effect of rudders;
- the effect of oars;
- the construction of rowed ships;
- the resistance of water to moved bodies;
- the force exerted by the wind on a sail;
- masting of sailing ships;
- a ship on a skew course.

Later Euler was worried that the above work could not be understood by ordinary seamen. Therefore, in 1773 he published a work that could be understood by laymen. It was entitled *Complete theory of construction and piloting of ships as to be applied to those who navigate*.

## 4.3 Lunar Motion

Calculating the orbit of the moon is much more difficult than calculating the orbit of the earth around the sun. In calculating the earth's orbit it is only necessary to consider the gravitational attraction between the earth and the sun. To first order the attraction of the other planets and the moon are negligible. However, for the moon, both the earth's attraction and the sun's attraction are important. Euler published a number of articles on this subject. Two of the longer publications were *Theory of the motion of the moon which exhibits all its irregularities*, 1753 (355 pages) and *Theory of lunar motion*, 1772 (791 pages). These publications contain results that involved a tremendous amount of calculation. The *Theory of lunar motion* has one table that is 144 pages long. Euler won the first prize of the Paris Academy of Sciences in 1770 and 1772 with papers on Lunar motion. Tobias Mayer used Euler's formulas to calculate Lunar tables for use in the calculation of Longitude. The Board of Longitude awarded 500 pounds to Mayer and 300 pounds to Euler for their contribution to navigation.

## 4.4 Analysis and Mechanics

I have lumped analysis and mechanics together since analysis is the key to understanding mechanics and mechanics provided most of the problems in analysis that mathematicians in the eighteenth century addressed. Over one half of the pages Euler published were either expressly devoted to mechanics or involved closely related topics. [7]. Newton and Leibnitz invented calculus in the seventeenth century, but it was Euler in the eighteenth century that developed it to the point where it was useful in solving physical problems. He showed that Newton's laws and other laws of physics could be formulated in terms of differential equations. He also investigated techniques for solving these differential equations. In particular, he pioneered the use of power series expansions. Euler was the first to publish a paper on partial differential equations. He also published the first textbook on calculus that could in any sense be considered complete. Historian Carl Boyer calls it the most important textbook of modern times. Here is a quote from his article *The Foremost Textbook of Modern Times (1950)*[1].

> *The most influential mathematics textbook of ancient times is easily named, for the Elements of Euclid has set the pattern in elementary geometry ever since. The most effective textbook of the medieval age is less easily designated; but a good case can be made out for the Al-jabr of Al-Khwarizmi, from which algebra arose and took its name. Is it possible to indicate a modern textbook of comparable influence and prestige? Some would mention the Géométrie of Descartes or the Principia of Newton or the Disquisitiones of Gauss; but in pedagogical significance these classics fell short of a work by Euler titled Introductio in analysin infinitorum. Here in effect Euler accomplished for analysis what Euclid and Al-Khwarizmi had done for synthetic geometry and elementary algebra respectively. Coordinate geometry, the function concept, and the calculus had arisen by the seventeenth century; yet it was the Introductio which in 1748 fashioned these into the third member of the triumvirate — comprising geometry, algebra, and analysis.*

> *. . . Euler avoided the phrase analytic geometry, probably to obviate confusion with the older Platonic usage; yet the second volume of the Introductio has been referred to, appropriately, as the first textbook on the subject. It contains the earliest systematic graphical study of functions of one and two independent variables, including the recognition of the quadrics as constituting a single family of surfaces. The Introductio was first also in the algorithmic treatment of logarithms as exponents and in the analytic treatment of the trigonometric functions as numerical ratios.*
>
> *The Introductio does not boast an impressive number of editions, yet its influence was pervasive. In originality and in the richness of its scope it ranks among the greatest of textbooks; but it is outstanding also for clarity of exposition. Published two hundred and two years ago, it nevertheless possesses a remarkable modernity of terminology and notation, as well as of viewpoint. Imitation is indeed the sincerest form of flattery.*

Euler did fundamental work dealing with the gravitational interaction of point masses, the motion of rigid bodies, and with elastic fluids and solids. His publications in these areas are too numerous to list.

## 4.5 Hydrodynamics and Hydraulics

Euler published the first general treatise on hydrodynamics. It consisted of three parts. The first paper entitled *Principles of the motion of fluids* was published in 1761. The second entitled *Section two of Principles of Fluid Motion* was first presented to the Saint Petersburg Academy in 1766. The third entitled *The third chapter on the linear motion of fluids, especially of water* was also presented in 1766. Euler's treatise on hydrodynamics contained the general equations for the motion of an ideal fluid (called Euler's equation), the conservation of mass (Equation of Continuity), and the conservation of fluid energy. It would be another 100 years before another treatise on hydrodynamics was published.

In hydraulics, Euler was the first to provide a complete theory of fluid driven turbines. His most in-depth treatment of the subject is contained in the paper *Complete theory of machines activated by water*, 1756. His treatment is so complete that an engineer today could use it to design a turbine.

## 4.6 Insurance

Euler wrote several fundamental papers in actuarial science that is the foundation of the insurance industry. One of these is entitled *General investigations on the mortality and the multiplication of the human race*, 1767. In it he considers such topics as

- A certain number of men, of whom all are the same age, being given, to find how many of them are probably yet alive after a certain number of years.

- To find the probability that a man of a certain age be still alive after a certain number of years.
- One demands that probability that a man of a certain age will die in the course of a given year.
- To find the term in which a man of a given age is able to hope to survive, of the kind that it is equally probable that he die before this term as after.
- To determine the life annuity that it is just to pay to a man of any age all the years, until his death, for a sum which will have been advanced first.
- When the interested parties are some infants newly born and when the payment of the life annuities must begin only when they will have attained a certain age, to determine the amount of these life annuities.

## 4.7 The Calculus of Variations

In calculus it is shown that a smooth curve has zero slope at places where it has a maximum or a minimum. The slope of the function defining a curve is called the derivative. Thus, the derivative of a smooth function is zero at points where it achieves an extreme value. The calculus of variations began as a generalization of this concept. Here the maximum or minimum is over a set of functions rather than over a set of points. For example, Fermat (1601–1665) studied optics problems involving more than one medium with different light speeds. He found that light follows the path requiring the least travel time. Here the travel time is minimized over all possible paths. A generalization of the derivative, called the variation, was defined for problems of this type that is zero for the desired optimum. It turns out that virtually all the laws of Physics can be expressed in variational form, i.e., in a form in which some scalar quantity has zero variation at the desired solution. Although the calculus of variations started out looking at maximums and minimums, the condition that the variation is zero over a class of function may not correspond to a maximum or minimum. The calculus of variations played a big role in the development of Einsteins general theory of relativity. Leonhard Euler was responsible for much of the early development in this field. Historians tell us that the motivation for the early development of the calculus of variations came from a belief in a God who designed the physical world in an efficient and optimal manner. Euler began by considering geodesic problems, i.e., finding the curve joining two points on a surface having minimum length. In 1728 he wrote a paper entitled *On finding the equation of geodesic curves*. Later, in 1744, he published a more general work entitled *A method for discovering curved lines that enjoy a maximum or minimum property, or the solution of the isoperimetric problem taken in the widest sense*. One of the things he considered in this paper was the minimization of integrals of the form

$$I = \int_a^b f(x, y, y')\, dx.$$

He showed that a necessary condition for a minimum was the Euler equation

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y}\right) = 0.$$

Euler also applied variational methods to problems in mechanics. The application of variational methods to mechanics began with the principle of least action. The credit for this principle is usually given to the French mathematician Maupertuis (1744), but was probably discovered earlier by Euler. It appeared in Euler's 1744 paper cited above. In the form developed by Euler, the action was the time integral of twice the kinetic energy (then known as the living force) over the path. The principle states that the actual path minimizes the action. It is now known that the principle in this form only holds for conservative systems, i.e., systems for which the sum of the kinetic energy and the potential energy is a constant.

## 4.8 Topological problems

Euler was very interested in geometric problems in which the concept of distance was not involved. He used the term geometry of position to describe such problems. Today we would include such problems under topology. An example of this type of problem is the Königsberg bridge problem. In the eighteenth century Königsberg was a city in Germany. It is now called Kaliningrad and is located in Russia. Figure 6 shows what present day Kaliningrad looks like. The Pregel river passed through Königsberg forming two islands as is shown in Figure 7a. There were seven bridges crossing the river. The problem was to see if there was a path a person could take that crossed each bridge exactly once and returned to the original starting point. Euler solved this problem in 1736, showing that it was not possible. It is often incorrectly stated that Euler solved this problem using graph theory [4]. It is true that today this problem is usually solved by making a graph as shown in Figure 7b.

Figure 6: Present day Kaliningrad

The nodes of the graph represent the four land masses. The lines represent the connections (bridges) between the land masses. It is easy to see that in this problem a successful path requires an even number of lines terminating at each node. For every pathway entering a region there must be a different pathway leaving the region. Since there are an odd number of pathways

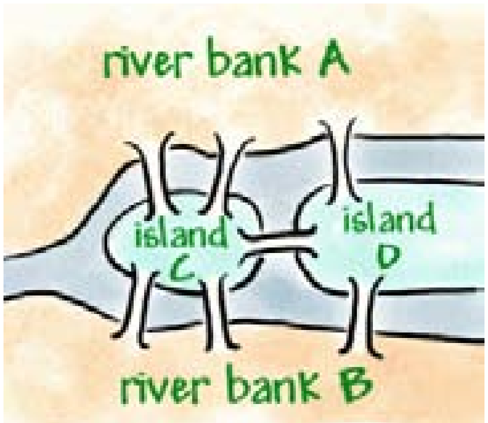


Figure 7a: Königsberg bridges

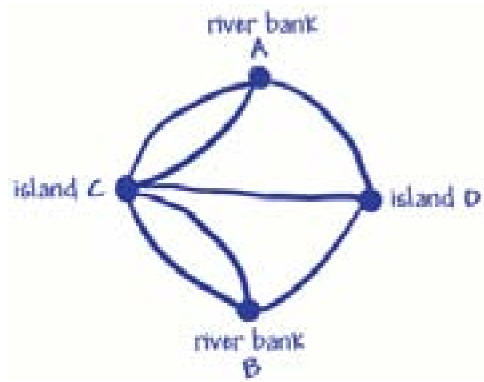


Figure 7b: A graph of the problem.

terminating at each node, there is no solution. If we don't require the desired path to start and end at the same point, then there could be two nodes at which an odd number of pathways terminated. In this problem there is no solution of this type either.

Euler didn't use graphs in his solution. He described a path in terms of the land masses visited. Thus a path was a sequence letters made up from the letters A, B, C, and D. Since there are seven bridges and each bridge can only be crossed once, a successful path must consist of eight letters. He then looked at how many times each of the four letters must be repeated. Since region A has three bridges, a path crossing each bridge exactly once would result in the letter A occurring twice. Similarly, the letters B and D must each occur twice. Since the region C has five bridges, the letter C must occur three times. Adding up the occurrences of the four letters we get nine. However, a path can only have eight letters. Therefore, there is no solution to this problem. Euler generalized this problem to more complicated problems having more regions and bridges. He also considered problems where the initial and final points are not the same.

The preceding sections give us a small sampling of the many areas in which Leonhard Euler made significant contributions. He was truly a great mathematician, a great physical scientist, and a great man. For further reading I would recommend the article *Leonard Euler, Supreme Geometer* by Clifford Truesdell , the lecture *Leonhard Euler: His Life, the Man, and His Works* by Walter Gautschi, and the paper *The God-Fearing Life of Leonhard Euler* by Dale McIntyre. These are listed in the references below.

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